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SCHOOL OF ENGINEERING

TECHNICAL REPORT

**SCATTERING OF OBLIQUELY INCIDENT LIGHT WAVES
BY ELLIPTICAL FIBERS**

Cavour W. H. Yeh

ELECTRICAL ENGINEERING DEPARTMENT

USC
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ABSTRACT

The exact solution of the scattering of obliquely incident plane waves by an elliptical dielectric cylinder is obtained. It is found that each expansion coefficient of the scattered or transmitted wave is coupled to all coefficients of the series expansion for the incident wave except when the elliptical cylinder degenerates to a circular one. Both polarizations of the incident wave are considered: one with the incident electric vector in the axial direction, and the other with the incident magnetic vector in the axial direction. It is noted that in the general case of oblique incidence the scattered field contains a significant cross-polarized component which vanishes at normal incidence.

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I. INTRODUCTION

The problems of scattering of waves by a circular dielectric cylinder have been considered by many authors (Refs. 1-3). Most recently, the exact solution of the problem of diffraction of normally incident plane waves by a dielectric elliptical cylinder was obtained (Ref. 4). However, the general case for oblique incidence has not been considered. It is the purpose of this paper to present a complete solution for this general problem. It is hoped that the results will be applicable to the problem of scattering of light by noncircular fibers.

II. FORMULATION OF THE PROBLEM

To analyze this problem, the elliptical cylinder coordinates (ξ, η, z) , as shown in Fig. 1, are introduced. In terms of the rectangular coordinates (x, y, z) , the elliptical cylinder coordinates are defined by the following relations:

$$\begin{aligned}x &= q \cosh \xi \cos \eta \\y &= q \sinh \xi \sin \eta \\z &= z\end{aligned}\tag{1}$$
$$(0 \leq \xi \leq \infty, 0 \leq \eta \leq 2\pi)$$

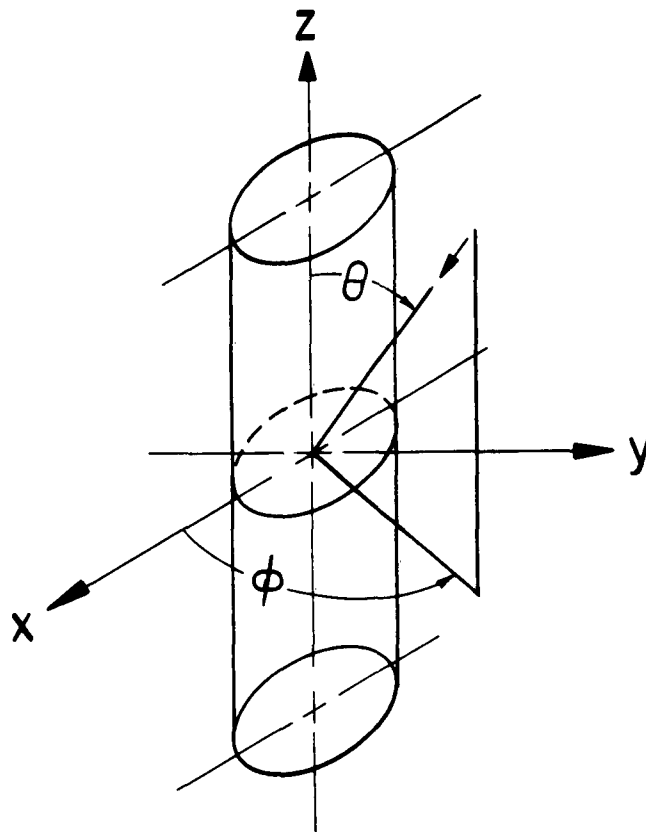


Fig. 1 Plane Wave Incident On An Elliptical Dielectric Cylinder. Arrow Indicates The Direction Of Incident Wave.

where q is the semifocal length of the ellipse. The contour surfaces of constant ξ , are confocal elliptic cylinders, and those of constant η are confocal hyperbolic cylinders. One of the confocal elliptic cylinders with $\xi = \xi_0$ is assumed to coincide with the boundary of the solid dielectric cylinder, and z -axis coincides with its longitudinal axis. It will be assumed that this dielectric cylinder, which has a permittivity ϵ_1 , a permeability μ_1 , and a conductivity of zero, is embedded in a homogeneous

perfect dielectric medium (ϵ_0, μ_0). A possible solution of the wave equation is then $R(\xi) \theta(\eta) e^{i\beta z} e^{-i\omega t}$ where R and θ satisfy the differential equations

$$\frac{d^2 R}{d\xi^2} - (c - 2\gamma^2 \cosh 2\xi) R = 0 \quad (2)$$

$$\frac{d^2 \theta}{d\eta^2} + (c - 2\gamma^2 \cos 2\eta) \theta = 0 \quad (3)$$

in which c is the separation constant and $\gamma^2 = (k^2 - \beta^2) q^2/4$, k being the wave number.

The periodic solutions of the equation in η are of two types: even about $\eta = 0$, and odd about $\eta = 0$ (Ref. 5). They are possible only for certain characteristic values of c . The even and odd functions are respectively denoted by $ce_n(\eta, \gamma^2)$ and $se_n(\eta, \gamma^2)$ with the sequence in n according to increasing values of c . It should be noted that these functions are orthogonal functions. The solutions of (2) corresponding to the even function $ce_n(\eta, \gamma^2)$ having the same characteristic values of c are $Ce_n(\xi, \gamma^2)$ and $Fey_n(\xi, \gamma^2)$, and those corresponding to the odd function $se_n(\eta, \gamma^2)$ are $Sen(\xi, \gamma^2)$ and $Gey_n(\xi, \gamma^2)$ (Ref. 5).

The proper choice of these functions to represent the electromagnetic fields depends upon the boundary conditions. For the region within the dielectric cylinder, all field components must be finite. All field components for the scattered wave must satisfy the Sommerfeld's

radiation condition at infinity. Consequently, the appropriate solutions of the wave equation for the region inside the dielectric cylinder are

$$\begin{pmatrix} C e_n(\xi, \gamma_1^2) & c e_n(\eta, \gamma_1^2) \\ S e_n(\xi, \gamma_1^2) & s e_n(\eta, \gamma_1^2) \end{pmatrix} e^{i\beta z} \quad (4)$$

and those for the scattered wave are

$$\begin{pmatrix} M e_n^{(1),(2)}(\xi, \gamma_0^2) & c e_n(\eta, \gamma_0^2) \\ N e_n^{(1),(2)}(\xi, \gamma_0^2) & s e_n(\eta, \gamma_0^2) \end{pmatrix} e^{i\beta z} \quad (5)$$

where

$$M e_n^{(1),(2)}(\xi, \gamma_0^2) = C e_n(\xi, \gamma_0^2) \pm i F e y_n(\xi, \gamma_0^2) \quad (6)$$

$$N e_n^{(1),(2)}(\xi, \gamma_0^2) = S e_n(\xi, \gamma_0^2) \pm i G e y_n(\xi, \gamma_0^2) \quad (7)$$

γ_1^2 and γ_0^2 are given in the next section.

III. SCATTERING OF AN OBLIQUELY INCIDENT PLANE WAVE

Two types of incident waves are possible. The one, called an E-wave is defined by $H_z = 0$, and the other called an H-wave is defined by $E_z = 0$.

It can be shown that a plane wave with its direction of propagation defined by the angles ϕ and θ (see Fig. 1) is given by

$$\begin{aligned}
& e^{ik_0(x \cos \phi \sin \theta + y \sin \phi \sin \theta + z \cos \theta)} \\
& = e^{ik_0 \left[q \sin \theta (\cosh \xi \cos \eta \cos \phi + \sinh \xi \sin \eta \sin \phi) + z \cos \theta \right]} \\
& = 2 \sum_{n=0}^{\infty} \left[\frac{1}{p_{2n}} Ce_{2n}(\xi, \gamma_0^2) ce_{2n}(\eta, \gamma_0^2) ce_{2n}(\phi, \gamma_0^2) \right. \\
& \quad + \frac{1}{s_{2n+2}} Se_{2n+2}(\xi, \gamma_0^2) se_{2n+2}(\eta, \gamma_0^2) se_{2n+2}(\phi, \gamma_0^2) \\
& \quad + \frac{i}{p_{2n+1}} Ce_{2n+1}(\xi, \gamma_0^2) ce_{2n+1}(\eta, \gamma_0^2) ce_{2n+1}(\phi, \gamma_0^2) \\
& \quad \left. + \frac{i}{s_{2n+1}} Se_{2n+1}(\xi, \gamma_0^2) se_{2n+1}(\eta, \gamma_0^2) se_{2n+1}(\phi, \gamma_0^2) \right] e^{ik_0 z \cos \theta}
\end{aligned} \tag{8}$$

where $\gamma_0^2 = k_0^2 q^2 \sin^2 \theta / 4$, $k_0^2 = 2\pi / \lambda_0$, and λ_0 is the free-space wavelength. p_{2n} , p_{2n+1} , s_{2n+2} , and s_{2n+1} are joining factors (Ref. 5).

In order to simplify the notations for the Mathieu and modified Mathieu functions without any ambiguities, the following abbreviations are used:

$$\begin{aligned}
Ce_n(\xi, \gamma_0^2) &= Ce_n(\xi) & ce_n(\eta, \gamma_0^2) &= ce_n(\eta) \\
Se_n(\xi, \gamma_0^2) &= Se_n(\xi) & se_n(\eta, \gamma_0^2) &= se_n(\eta) \\
Ce_n(\xi, \gamma_1^2) &= Ce_n^*(\xi) & ce_n(\eta, \gamma_1^2) &= ce_n^*(\eta) \\
Se_n(\xi, \gamma_1^2) &= Se_n^*(\xi) & se_n(\eta, \gamma_1^2) &= se_n^*(\eta)
\end{aligned}$$

$$Me_n^{(1),(2)}(\xi, \gamma_0^2) = Me_n^{(1),(2)}(\xi) \quad (9)$$

$$Ne_n^{(1),(2)}(\xi, \gamma_0^2) = Ne_n^{(1),(2)}(\xi)$$

with $\gamma_1^2 = (k_1^2 - k_0^2 \cos^2 \theta) q^2/4$ where $k_1^2 = \omega^2 \mu_1 \epsilon_1$.

A. E-WAVE

The axial components of an incident E-wave are

$$E_z^i = E_0 \left\{ \text{the right-hand side of Eq. (8)} \right\} \quad (10)$$

$$H_z^i = 0. \quad (11)$$

It is interesting to note that, unlike the case for a normally incident wave or for a perfectly conducting cylinder at oblique incidence, the boundary conditions for the present general case of oblique incidence on a dielectric elliptical cylinder cannot be satisfied if the z component of the scattered or transmitted magnetic field is taken to be zero for an incident E-wave, or if the z component of the scattered or transmitted electric field is taken to be zero for an incident H-wave. Hence, referring to (4) and (5), we see that the axial components of the scattered field and of the transmitted field inside the dielectric cylinder must be of the form

$$\begin{aligned}
E_z^s = 2E_0 \sum_{n=0}^{\infty} & \left[\frac{A_{2n}}{p_{2n}} Me_{2n}^{(1)}(\xi) ce_{2n}(\eta) ce_{2n}(\phi) \right. \\
& + \frac{B_{2n+2}}{s_{2n+2}} Ne_{2n+2}^{(1)}(\xi) se_{2n+2}(\eta) se_{2n+2}(\phi) \\
& + i \frac{A_{2n+1}}{p_{2n+1}} Me_{2n+1}^{(1)}(\xi) ce_{2n+1}(\eta) ce_{2n+1}(\phi) \\
& \left. + i \frac{B_{2n+1}}{s_{2n+1}} Ne_{2n+1}^{(1)}(\xi) se_{2n+1}(\eta) se_{2n+1}(\phi) \right] e^{ik_0 z \cos \theta}
\end{aligned}$$

(12)

$$\begin{aligned}
H_z^s = 2E_0 \sum_{n=0}^{\infty} & \left[\frac{C_{2n}}{p_{2n}} Me_{2n}^{(1)}(\xi) ce_{2n}(\eta) ce_{2n}(\phi) \right. \\
& + \frac{D_{2n+2}}{s_{2n+1}} Ne_{2n+2}^{(1)}(\xi) se_{2n+2}(\eta) se_{2n+2}(\phi) \\
& + i \frac{C_{2n+1}}{p_{2n+1}} Me_{2n+1}^{(1)}(\xi) ce_{2n+1}(\eta) ce_{2n+1}(\phi) \\
& \left. + i \frac{D_{2n+1}}{s_{2n+1}} Ne_{2n+1}^{(1)}(\xi) se_{2n+1}(\eta) se_{2n+1}(\phi) \right] e^{ik_0 z \cos \theta}
\end{aligned}$$

(13)

and

$$\begin{aligned}
E_z^t = 2E_0 \sum_{n=0}^{\infty} & \left[\frac{F_{2n}}{p_{2n}^*} ce_{2n}^*(\xi) ce_{2n}^*(\eta) ce_{2n}(\phi) \right. \\
& + \frac{G_{2n+2}}{s_{2n+2}^*} se_{2n+2}^*(\xi) se_{2n+2}^*(\eta) se_{2n+2}(\phi)
\end{aligned}$$

$$\begin{aligned}
& + i \frac{F_{2n+1}}{p_{2n+1}^*} Ce_{2n+1}^*(\xi) ce_{2n+1}(\eta) ce_{2n+1}(\phi) \\
& + i \frac{G_{2n+1}}{s_{2n+1}^*} Se_{2n+1}^*(\xi) se_{2n+1}^*(\eta) se_{2n+1}(\phi) \Big] e^{ik_0 z \cos \theta}
\end{aligned}$$

(14)

$$\begin{aligned}
H_z^\dagger = 2E_0 \sum_{n=0}^{\infty} & \left[\frac{P_{2n}}{p_{2n}^*} Ce_{2n}^*(\xi) ce_{2n}^*(\eta) ce_{2n}(\phi) \right. \\
& + \frac{Q_{2n+2}}{s_{2n+2}^*} Se_{2n+2}^*(\xi) se_{2n+2}^*(\eta) se_{2n+2}(\phi) \\
& + i \frac{P_{2n+1}}{p_{2n+1}^*} Ce_{2n+1}^*(\xi) ce_{2n+1}^*(\eta) ce_{2n+1}(\phi) \\
& \left. + i \frac{Q_{2n+1}}{s_{2n+1}^*} Se_{2n+1}^*(\xi) se_{2n+1}^*(\eta) se_{2n+1}(\phi) \right] e^{ik_0 z \cos \theta}
\end{aligned}$$

(15)

where A_n , B_n , C_n , D_n , F_n , G_n , P_n , and Q_n are arbitrary unknown coefficients that can be determined by applying the boundary conditions. p_{2n}^* , p_{2n+1}^* , s_{2n+2}^* , and s_{2n+1}^* are joining factors. All transverse fields can be derived from Maxwell's equations with the knowledge of the axial fields.

The boundary conditions require the continuity of the tangential components of the electric and magnetic field at the boundary surface $\xi = \xi_0$; i.e.,

$$\begin{aligned}
& \left[\text{r. h. s. of Eq. (10) with } \xi = \xi_0 \right] + \left[\text{r. h. s. of Eq. (12) with } \xi = \xi_0 \right] \\
& = \left[\text{r. h. s. of Eq. (14) with } \xi = \xi_0 \right], \tag{16}
\end{aligned}$$

$$\begin{aligned}
& \left[\text{r. h. s. of Eq. (13) with } \xi = \xi_0 \right] = \left[\text{r. h. s. of Eq. (15) with } \xi = \xi_0 \right], \\
& \tag{17}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{k_0^2 \sin^2 \theta} \left\{ ik_0 \cos \theta \frac{\partial}{\partial \eta} \left[\text{r. h. s. of Eq. (10) with } \xi = \xi_0 \right] \right. \\
& \quad + ik_0 \cos \theta \frac{\partial}{\partial \eta} \left[\text{r. h. s. of Eq. (12) with } \xi = \xi_0 \right] \\
& \quad \left. - i\omega \mu_0 \frac{\partial}{\partial \xi_0} \left[\text{r. h. s. of Eq. (13) with } \xi = \xi_0 \right] \right\} \\
& = \frac{1}{(k_1^2 - k_0^2 \cos^2 \theta)} \left\{ ik_0 \cos \theta \frac{\partial}{\partial \eta} \left[\text{r. h. s. of Eq. (14) with } \xi = \xi_0 \right] \right. \\
& \quad \left. - i\omega \mu_1 \frac{\partial}{\partial \xi_0} \left[\text{r. h. s. of Eq. (15) with } \xi = \xi_0 \right] \right\}, \tag{18}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{k_0^2 \sin^2 \theta} \left\{ -i\omega \epsilon_0 \frac{\partial}{\partial \xi_0} \left[\text{r. h. s. of Eq. (10) with } \xi = \xi_0 \right] \right. \\
& \quad - i\omega \epsilon_0 \frac{\partial}{\partial \xi_0} \left[\text{r. h. s. of Eq. (12) with } \xi = \xi_0 \right] \\
& \quad \left. - ik_0 \cos \theta \frac{\partial}{\partial \eta} \left[\text{r. h. s. of Eq. (13) with } \xi = \xi_0 \right] \right\} \\
& = \frac{1}{(k_1^2 - k_0^2 \cos^2 \theta)} \left\{ -i\omega \epsilon_1 \frac{\partial}{\partial \xi_0} \left[\text{r. h. s. of Eq. (14) with } \xi = \xi_0 \right] \right. \\
& \quad \left. - ik_0 \cos \theta \frac{\partial}{\partial \eta} \left[\text{r. h. s. of Eq. (15) with } \xi = \xi_0 \right] \right\}, \tag{19}
\end{aligned}$$

where r.h.s. means the right-hand side. It is noted that in contrast with the spherical or circular cylinder case the angular functions in the elliptical cylinder case are functions not only of the angular component but also of the characteristics of the medium. Consequently, the summation signs and the angular functions in the above equations may not be omitted. However, it will be shown that this difficulty may be overcome by the orthogonality properties of Mathieu functions. Substituting the expansions

$$ce_m^*(\eta) = \sum_{n=0}^{\infty} \alpha_{m,n} ce_n(\eta) \quad (20a)$$

$$se_m^*(\eta) = \sum_{n=0}^{\infty} \beta_{m,n} se_n(\eta) \quad (20b)$$

$$\frac{d}{d\eta} ce_m(\eta) = \sum_{n=0}^{\infty} \gamma_{m,n} se_n(\eta) \quad (20c)$$

$$\frac{d}{d\eta} se_m(\eta) = \sum_{n=0}^{\infty} \chi_{m,n} ce_n(\eta) \quad (20d)$$

into Eqs. (16) through (19), and applying the orthogonality relations of Mathieu functions, leads to the following expressions:

$$\frac{1}{p_n} \left[ce_n(\xi_0) + A_n Me_n^{(1)}(\xi_0) \right] ce_n(\phi) = \sum_{m=0}^{\infty} \frac{F_m}{p_m^*} ce_m^*(\xi_0) ce_m(\phi) \alpha_{m,n}, \quad (21)$$

$$\frac{1}{s_n} \left[\text{Se}_n(\xi_0) + B_n \text{Ne}_n^{(1)}(\xi_0) \right] \text{se}_n(\phi) = \sum_{m=0}^{\infty} \frac{G_m}{s_m^*} \text{Se}_m^*(\xi_0) \text{se}_m^*(\phi) \beta_{m,n}, \quad (22)$$

$$\frac{C_n}{p_n} \text{Me}_n^{(1)}(\xi_0) \text{ce}_n(\phi) = \sum_{m=0}^{\infty} \frac{P_m}{p_m^*} \text{Ce}_m^*(\xi_0) \text{ce}_m(\phi) \alpha_{m,n}, \quad (23)$$

$$\frac{D_n}{s_n} \text{Ne}_n^{(1)}(\xi_0) \text{se}_n(\phi) = \sum_{m=0}^{\infty} \frac{Q_m}{s_m^*} \text{Se}_m^*(\xi_0) \text{se}_m(\phi) \beta_{m,n}, \quad (24)$$

$$\sqrt{\frac{\epsilon_0}{\mu_0}} \cos \theta \left(1 - \frac{k_0^2 \sin^2 \theta}{k_1^2 - k_0^2 \cos^2 \theta} \right) \left[\sum_{m=0}^{\infty} \frac{1}{s_m} \left[\text{Se}_m(\xi_0) + B_m \text{Ne}_m^{(1)}(\xi_0) \right] \text{se}_m(\phi) \chi_{m,n} \right]$$

$$- \frac{C_n}{p_n} \text{Me}_n^{(1)'}(\xi_0) \text{ce}_n(\phi)$$

$$= \left(-\frac{\mu_1}{\mu_0} \right) \left(\frac{k_0^2 \sin^2 \theta}{k_1^2 - k_0^2 \cos^2 \theta} \right) \sum_{m=0}^{\infty} \frac{P_m}{p_m^*} \text{Ce}_m^*(\xi_0) \text{ce}_m(\phi) \alpha_{m,n}, \quad (25)$$

$$\sqrt{\frac{\epsilon_0}{\mu_0}} \cos \theta \left(1 - \frac{k_0^2 \sin^2 \theta}{k_1^2 - k_0^2 \cos^2 \theta} \right) \left[\sum_{m=0}^{\infty} \frac{1}{p_m} \left[\text{Ce}_m(\xi_0) + A_m \text{Me}_m^{(1)}(\xi_0) \right] \text{ce}_m(\phi) \gamma_{m,n} \right]$$

$$- \frac{D_n}{s_n} \text{Ne}_n^{(1)'}(\xi_0) \text{se}_n(\phi)$$

$$= \left(-\frac{\mu_1}{\mu_0} \right) \frac{k_0^2 \sin^2 \theta}{k_1^2 - k_0^2 \cos^2 \theta} \sum_{m=0}^{\infty} \frac{Q_m}{s_m^*} \text{Se}_m^*(\xi_0) \text{se}_m(\phi) \beta_{m,n}, \quad (26)$$

$$\begin{aligned}
& \frac{1}{s_n} \left[Se_n'(\xi_0) + B_n Ne_n^{(1)'}(\xi_0) \right] se_n(\phi) \\
& + \sqrt{\frac{\mu_0}{\epsilon_0}} \cos \theta \left(1 - \frac{k_0^2 \sin^2 \theta}{k_1^2 - k_0^2 \cos^2 \theta} \right) \sum_{m=0}^{\infty'} \frac{C_m}{p_m} Me_m^{(1)}(\xi_0) ce_m(\phi) \gamma_{m,n} \\
& = \frac{k_0^2 \sin^2 \theta}{k_1^2 - k_0^2 \cos^2 \theta} \frac{\epsilon_1}{\epsilon_0} \sum_{m=0}^{\infty'} \frac{G_m}{s_m^*} Se_m^*(\xi_0) se_m(\phi) \beta_{m,n} \quad ,
\end{aligned} \tag{27}$$

$$\begin{aligned}
& \frac{1}{p_n} \left[Ce_n'(\xi_0) + A_n Me_n^{(1)'}(\xi_0) \right] ce_n(\phi) \\
& + \sqrt{\frac{\mu_0}{\epsilon_0}} \cos \theta \left(1 - \frac{k_0^2 \sin^2 \theta}{k_1^2 - k_0^2 \cos^2 \theta} \right) \sum_{m=0}^{\infty'} \frac{D_m}{s_m} Ne_m^{(1)}(\xi_0) se_m(\phi) \chi_{m,n} \\
& = \frac{k_0^2 \sin^2 \theta}{k_1^2 - k_0^2 \cos^2 \theta} \frac{\epsilon_1}{\epsilon_0} \sum_{m=0}^{\infty'} \frac{F_m}{p_m^*} Ce_m^*(\xi_0) ce_m(\phi) \alpha_{m,n} \quad .
\end{aligned} \tag{28}$$

$$(n = 0, 1, 2, 3, 4, \dots).$$

$\alpha_{m,n}, \beta_{m,n}, \gamma_{m,n}$, and $\chi_{m,n}$ are given in the Appendix. The prime on the summation sign means that when n is odd, the above series are summed over all odd values of m , and when n is even, the series are summed over all even values of m . The primes on the modified Mathieu functions denote differentiation with respect to ξ_0 . The unknown coefficients $A_n, B_n, C_n, D_n, F_n, G_n, P_n$, and Q_n can now be obtained from the above equations. Combining Eqs. (21), (24), (26), and (28) gives

$$\sum_{m=0}^{\infty} F_m s_{mn}^e + Q_m t_{mn}^e = 0 \quad (29)$$

$$\sum_{m=0}^{\infty} F_m u_{mn}^e + Q_m v_{mn}^e = w_n^e \quad (30)$$

$$(n=0, 1, 2, 3 \dots)$$

and combining Eqs. (22), (23), (25) and (27) gives

$$\sum_{m=0}^{\infty} G_m s_{mn}^0 + P_m t_{mn}^0 = 0 \quad (31)$$

$$\sum_{m=0}^{\infty} G_m u_{mn}^0 + P_m v_{mn}^0 = w^0 \quad (32)$$

$$(n=0, 1, 2, 3 \dots)$$

where

$$s_{mn}^e = \sqrt{\frac{\epsilon_0}{\mu_0}} \cos \theta (1 - x^2) f_m \sum_{r=0}^{\infty} a_{mr} \gamma_{mn}$$

$$s_{mn}^0 = \sqrt{\frac{\epsilon_0}{\mu_0}} \cos \theta (1 - x^2) g_m \sum_{r=0}^{\infty} \beta_{mr} \chi_{mn}$$

$$t_{mn}^e = \left[\frac{\mu_1}{\mu_0} x^2 g'_m - \frac{b'_n}{b_n} g_m \right] \beta_{mn}$$

$$t_{mn}^0 = \left[\frac{\mu_1}{\mu_0} x^2 h'_m - \frac{a'_n}{a_n} h_m \right] \alpha_{mn}$$

$$u_{mn}^e = \left[f_m \frac{a_n'}{a_n} - x^2 \frac{\epsilon_1}{\epsilon_0} f_m' \right] \alpha_{mn}$$

$$u_{mn}^o = \left[g_m \frac{b_n'}{b_n} - x^2 \frac{\epsilon_1}{\epsilon_0} g_m' \right] \beta_{mn}$$

$$v_{mn}^e = \sqrt{\frac{\mu_0}{\epsilon_0}} \cos \theta (1-x^2) g_m \sum_{r=0}^{\infty} \beta_{mr} \chi_{rn}$$

$$v_{mn}^o = \sqrt{\frac{\mu_0}{\epsilon_0}} \cos \theta (1-x^2) f_m \sum_{r=0}^{\infty} \alpha_{mr} \gamma_{rn}$$

$$w_n^o = h_n \frac{a_n'}{a_n} - h_n'$$

$$w_n^e = g_n \frac{b_n'}{b_n} - g_n'$$

(33)

The following abbreviations have been used:

$$h_n = \frac{1}{p_n} C e_n(\xi_0) c e_n(\phi),$$

$$h_n' = \frac{1}{p_n} C e_n'(\xi_0) c e_n(\phi),$$

$$g_n = \frac{1}{s_n} S e_n(\xi_0) s e_n(\phi),$$

$$g_n' = \frac{1}{s_n} S e_n'(\xi_0) s e_n(\phi),$$

$$a_n = \frac{1}{p_n} M e_n^{(1)}(\xi_0) c e_n(\phi),$$

$$a_n' = \frac{1}{p_n} M e_n^{(1)'}(\xi_0) c e_n(\phi),$$

$$b_n = \frac{1}{s_n} N e_n^{(1)}(\xi_0) s e_n(\phi),$$

$$b_n' = \frac{1}{s_n} N e_n^{(1)'}(\xi_0) s e_n(\phi),$$

$$f_n = \frac{1}{p_n^*} C e_n^*(\xi_0) c e_n(\phi), \quad f_n' = \frac{1}{p_n^*} C e_n^{*'}(\xi_0) c e_n(\phi),$$

$$g_n = \frac{1}{s_n^*} S e_n^*(\xi_0) s e_n(\phi), \quad g_n' = \frac{1}{s_n^*} S e_n^{*'}(\xi_0) s e_n(\phi),$$

$$x = \frac{k_0^2 \sin^2 \theta}{k_1^2 - k_0^2 \cos^2 \theta} \quad (34)$$

The coefficients F_m and Q_m , and G_m and P_m can be obtained readily from Eqs. (29) and (30), and Eqs. (31) and (32) respectively. Knowing F_m , Q_m , G_m , and P_m , the coefficients A_n , B_n , C_n , and D_n can be found, respectively, from Eqs. (21) through (24).

It is interesting to note that the roots of the determinant of Eqs. (29) and (30) provide the propagation constants of a set of surface waves along an elliptical dielectric cylinder (Ref. 6). Due to the asymmetry of the elliptic cylinder, it is possible to have two orientations for the field configurations. The propagation constants of the other set of surface waves are obtained from the roots of the determinant of Eqs. (31) and (32). This surface wave problem has a direct application to the problem of guiding light waves along a flat fiber i. e., the fiber optics problem (Ref. 6) .

The transmitted fields in the dielectric cylinder and the scattered fields in free space due to an incident E-wave are now completely determined.

B. H-WAVE

The corresponding result for the case of an incident H-wave is of the same form as above if E is replaced by H , H is replaced by $-E$, ϵ is replaced by μ , and μ is replaced by ϵ , throughout.

IV. CONCLUSIONS

The exact solution of the problem of the scattering of an obliquely incident plane wave by an elliptical dielectric cylinder, or by a dielectric ribbon is obtained. It is noted that unlike the case for a circular dielectric cylinder, each expansion coefficient of the scattered or transmitted wave for the elliptical dielectric cylinder is coupled to all coefficients of the series expansion for the incident wave. Hence, the results are much more involved. Numerical investigation for the case of normally incident plane waves shows that the infinite determinants representing these expansion coefficients for the scattered or transmitted wave converge quite rapidly for small values of $k_0 q/2$; only the first few terms in the determinants are needed as long as $k_0 q/2$ is less than 5 (Ref. 4). This range of $k_0 q/2$ covers, however, most of the range not covered by the usual approximate diffraction theory. The results presented here are particularly useful in studying the scattering of light by thin fiber ribbons.

APPENDIX

FORMULAS FOR $\alpha_{m,n}$, $\beta_{m,n}$, $\gamma_{m,n}$, AND $\chi_{m,n}$

It can readily be shown from the theory of Mathieu functions that

$$\alpha_{m,n} = \int_0^{2\pi} ce_m^*(\eta) ce_n(\eta) d\eta / \int_0^{2\pi} ce_n^2(\eta) d\eta$$

$$\beta_{m,n} = \int_0^{2\pi} se_m^*(\eta) se_n(\eta) d\eta / \int_0^{2\pi} se_n^2(\eta) d\eta$$

$$\gamma_{m,n} = \int_0^{2\pi} ce_m'(\eta) sa_n(\eta) d\eta / \int_0^{2\pi} se_n^2(\eta) d\eta$$

$$\chi_{m,n} = \int_0^{2\pi} se_m'(\eta) ce_n(\eta) d\eta / \int_0^{2\pi} ce_n^2(\eta) d\eta$$

where the prime signifies the derivative of the function with respect to its argument. The above integrals can easily be integrated using the series expansions of Mathieu functions in terms of trigonometric functions (Ref. 5).

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<p>Both polarizations of the incident wave are considered: one with the incident electric vector in the axial direction, and the other with the incident magnetic vector in the axial direction. It is noted that in the general case of oblique incidence the scattered field contains a significant cross-polarized component which vanishes at normal incidence.</p>	<ol style="list-style-type: none"> VI. Aval fr OTS VII. In DDS collection 	<p>Both polarizations of the incident wave are considered: one with the incident electric vector in the axial direction, and the other with the incident magnetic vector in the axial direction. It is noted that in the general case of oblique incidence the scattered field contains a significant cross-polarized component which vanishes at normal incidence.</p>	<ol style="list-style-type: none"> VI. Aval fr OTS VII. In DDS collection